The comments concerning the annular rheometer should also apply to the falling cylinder rheometer since it is essentially an annular flow system with a moving boundary.

From the above discussion one may conclude that the method proposed by Swift et al. using the equations developed for the Couette, annular, and falling cylinder rheometers to obtain shear stress—shear rate data is an approximate one in which the accuracy of the results improve as $R_i \rightarrow R_o$.

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NOTATION

f_i = functionality symbol
L = length over which ΔP is measured
m = power law constant
m' = function of torque, T, defined by Equation (*2a*)
m'' = function of shear stress defined by Equation (*1a*)
n = power law constant
n' = function of torque, T, defined by Equation (*2a*)
n'' = function of shear stress defined by Equation (*1a*)
ΔP = pressure drop
Q = volumetric flow rate
Q_1 = volumetric flow rate when $\tau_w = 1.0$
r = radial position
R = capillary radius
R_i = radius of inner cylinder
R_o = radius of outer cylinder
T = torque
v_t = terminal velocity of falling cylinder

Greek Letters

$\dot{\gamma}$ = shear rate
λ = dimensionless radius of zero shear
ρ = fluid density
σ = density of falling body
τ = shear stress
Ω_o = angular velocity at outer wall

Subscripts

i = inner wall
o = outer wall
R = wall of capillary

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Mass and Heat Transfer from Rigid Spheres

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Keey and Glen (*11*) have presented a critical analysis of results of investigations of mass transfer from rigid spheres. They consider the assumption that an exponential expression of the form

$$N_{Sh} = A + B\, N_{Re}{}^{m}\, N_{Sc}{}^{n} \tag{1}$$

can be fitted to experimental data and point out that this equation is inadequate because the exponents m and n are not constants. This paper considers the variation of the exponents m and n with the Reynolds and Schmidt or Prandtl numbers as indicated by experimental heat and mass transfer data.

The constant A has been theoretically treated (*13*) and experimentally verified to equal 2.

REYNOLDS NUMBER RESPONSE

Figure 1 shows the Nusselt number as a function of Reynolds number at a constant Prandtl number of 0.73. Figure 2 shows the Sherwood number as a function of Reynolds number at a constant Schmidt number of 0.6 and Figure 3 shows the Sherwood number as a function of Reynolds number at a constant Schmidt number of 2,220.

Garner, Jenson, and Keey (*7*) have described the transitions in flow patterns that occur around spheres as a function of Reynolds number. At Reynolds numbers between 1 and 17 the flow is approximately symmetrical. At a Reynolds number of about 17 separation occurs and a weak toroidal vortex is formed near the rear stagnation point. As the Reynolds number increases, the vortex gains strength and the separation ring advances toward the equator, until at about a Reynolds number of 450 the separation angle is 104 deg. (*6*). The wake then becomes unstable and oscillates about the axis of motion and this continues above a Reynolds number of 1,000. The angle of separation remains at 104 deg. but the frequency of oscillation of the wake increased with Reynolds number. These two transition points are shown on the figures. Data

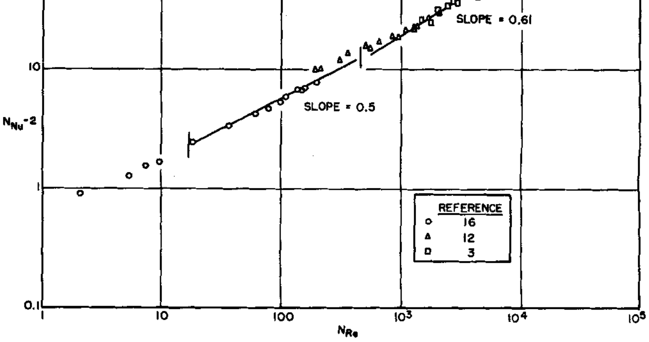

Fig. 1. Heat transfer data at $N_{Pr} = 0.73$.

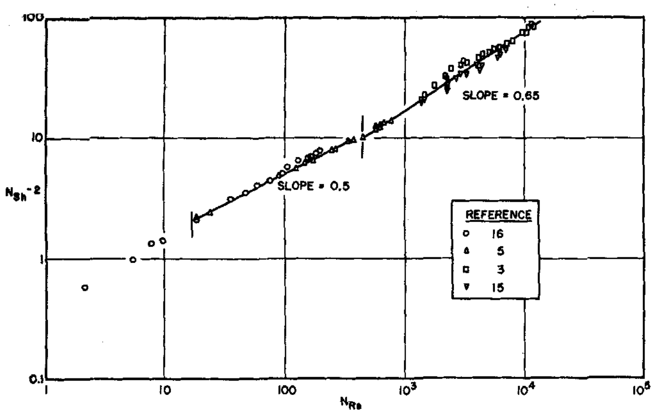

Fig. 2. Mass transfer data at $N_{Sc} = 0.6$.

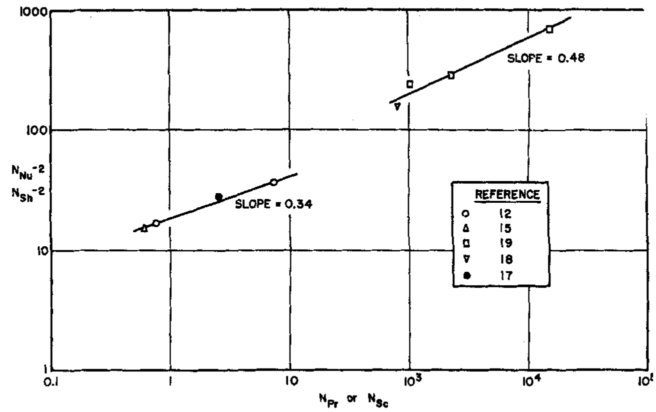

Fig. 6. Heat and mass transfer data at $N_{Re} = 1{,}000$.

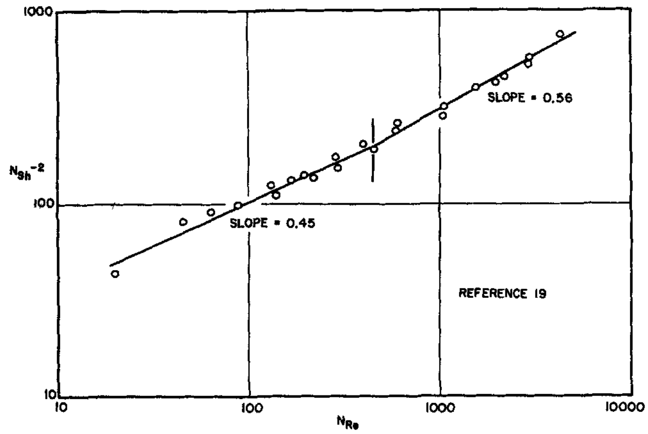

Fig. 3. Heat and mass transfer data at $N_{Sc} = 2{,}220$.

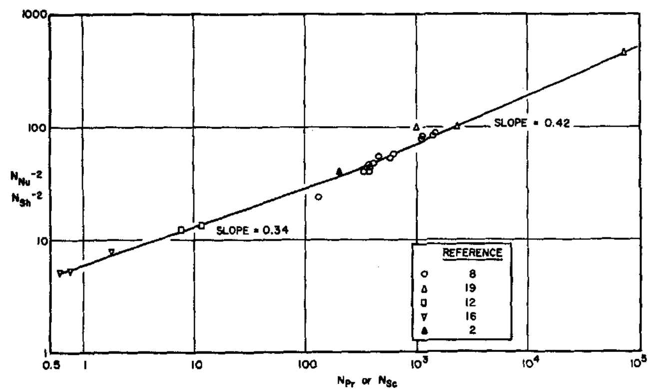

Fig. 4. Heat and mass transfer data at $N_{Re} = 100$.

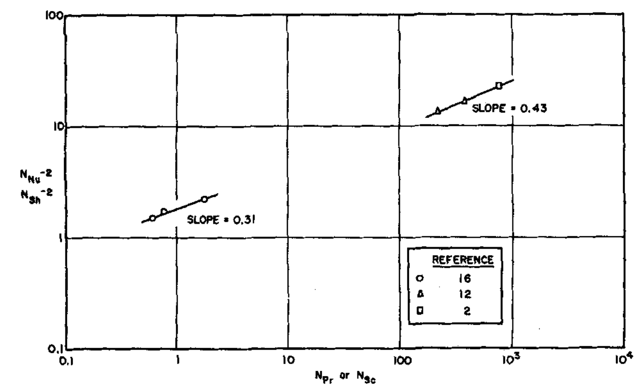

Fig. 5. Heat and mass transfer data at $N_{Re} = 10$.

below a Reynolds number of 450 show a slope of about 0.5 which is consistent with the film theory. At Reynolds numbers between 450 and 10,000 the slope appears to increase to about 0.62. This indicates an additional contribution to transfer from the oscillating wake.

SCHMIDT OR PRANDTL NUMBER RESPONSE

Figure 4 shows the Sherwood or Nusselt number as a function of the Schmidt or Prandtl number from smoothed experimental data at a Reynolds number of 100. It is apparent that the slope increases with increasing Schmidt or Prandtl numbers. These slopes can be approximated by the values of 0.33 at Schmidt or Prandtl numbers less than 250 and 0.42 above 250. It is noted that the data point for benzoic acid spheres in water is significantly higher than the correlating line. This high mass transfer rate may be caused by the relatively high solubility of benzoic acid in water which results in a rough surface as reported by Linton and Sherwood (*14*). Figures 5 and 6 show data for Reynolds numbers of 10 and 1,000. The slopes appear to be about the same as indicated by Figure 4.

Comparison of exponents for Reynolds and Prandtl numbers for heat transfer inside tubes shows a similar response to that for rigid spheres. Friend and Metzner (*4*) obtained experimental data for high Prandtl number fluids and found that the exponent on the Reynolds number increased with Prandtl number and that the exponent for the Prandtl numbers in the range of 20 to 350 was 0.42 at a constant Reynolds number of 10,000.

CORRELATIONS

Values of m and n obtained from the smoothed experimental data can then be used with the experimental data to obtain values of B. The resulting equations are

$1 < N_{Re} < 450,\quad N_{Sc}$ or $N_{Pr} < 250$

$$N_{Sh} \text{ or } N_{Nu} = 2 + 0.6\, N_{Re}^{1/2}\, (N_{Sc} \text{ or } N_{Pr})^{1/3} \qquad (2)$$

$1 < N_{Re} < 17,\quad 250 < N_{Sc}$ or N_{Pr}

$$N_{Sh} \text{ or } N_{Nu} = 2 + 0.5\, N_{Re}^{1/2}\, (N_{Sc} \text{ or } N_{Pr})^{0.42} \qquad (3)$$

TABLE 1. COMPARISON OF EXPERIMENTAL AND CALCULATED COEFFICIENTS

Equation	No. of Data Points	Average Absolute Deviation, %	Standard Deviation, %	Data References
2	208	8	6.6	2,5,10,12,16,17
3	43	11.2	8.0	2,12,19,21
4	45	9.5	7.7	2,8,19
5	174	10.3	9.9	1,3,5,12,15,20
6	33	9.3	7.7	18,19

$17 < N_{Re} < 450, \quad 250 < N_{Sc}$ or N_{Pr}

$$N_{Sh} \text{ or } N_{Nu} = 2 + 0.4\, N_{Re}^{1/2}\, (N_{Sc} \text{ or } N_{Pr})^{0.42} \quad (4)$$

$450 < N_{Re} < 10{,}000, \quad N_{Sc}$ or $N_{Pr} < 250$

$$N_{Sh} \text{ or } N_{Nu} = 2 + 0.27\, N_{Re}^{0.62}\, (N_{Sc} \text{ or } N_{Pr})^{1/3} \quad (5)$$

$450 < N_{Re} < 10{,}000, \quad 250 < N_{Sc}$

$$N_{Sh} = 2 + 0.175\, N_{Re}^{0.62}\, N_{Sc}^{0.42} \quad (6)$$

Table 1 shows the average absolute deviation and the standard deviation between experimental transfer coefficients and coefficients calculated from the equations.

Hammerton and Garner (9) obtained data for carbon dioxide bubbles in glycerine. Two of these data points could be expected to represent rigid sphere mass transfer. Extrapolation of Equation (3) shows an average absolute deviation of 27% for these data which represent a Schmidt number of 1.78×10^7.

SUMMARY

Smoothed heat and mass transfer data for rigid spheres have been plotted to investigate the exponents of the Reynolds and Schmidt numbers for Equation (1). The results indicate that the exponent for the Schmidt number increases with the Reynolds number and that the exponent for the Reynolds number indicates an eddy contribution from the wake at Reynolds numbers greater than 450.

NOTATION

C_p = specific heat, B.t.u./lb.$_m$ °F.
D_p = diameter of rigid sphere, ft.
$\mathfrak{D}$ = diffusivity, sq.ft./sec.
h = heat transfer coefficient, B.t.u./sec./sq.ft. °F.
k = thermal conductivity, B.t.u./sec./sq.ft.°F./ft.
k_c = mass transfer coefficient, ft./sec.
N_{Nu} = $h\, D_p/k$, Nusselt number
N_{Pr} = $C_p\, \mu/k$, Prandtl number
N_{Re} = $D_p\, V/\nu$, Reynolds number
N_{Sc} = $\nu/\mathfrak{D}$, Schmidt number
N_{Sh} = $k_c\, D_p/\mathfrak{D}$, Sherwood number
V = slip velocity, ft./sec.
μ = viscosity, lb.$_m$/ft./sec.
ν = kinematic viscosity, sq.ft./sec.

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Catalytic Hydrogenation of Olefins

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Hougen and co-workers (*1*) proposed the competitive-noncompetitive reaction rate model to correlate the data for the catalytic hydrogenation of propylene and isobutylene. Assuming that all of the catalytic sites are available to the smaller molecules, but that the larger molecules tend to exclude one another from adjacent sites through steric hindrance, and assuming that the reaction is surface reaction rate controlling, the kinetic analysis gives rise to a rate expression consisting of two terms, called the competitive and the noncompetitive terms which are as follows:

$$r = \frac{A K_H K_U \left(p_H p_U - \frac{p_S}{K} \right)}{(1 + K_H p_H + K_U p_U + K_S p_S)^2} + \frac{B K_H K_U \left(p_H p_U - \frac{p_S}{K} \right)}{(1 + K_H p_H)(1 + K_H p_H + K_U p_U + K_S p_S)} \quad (1)$$

For the hydrogenation of olefins, the equilibrium constant is large, and as long as the product concentration is